Charm Production in the DONuT Beam Dump

Introduction

The total and differential charm cross sections from proton-nucleon interactions are summarized from the most current sources that I could find. There is some new data available from Hera-B and SELEX. The Hera-B data is (mostly) published, but the SELEX data is not. Nevertheless I will use both to get the most likely values for $D^+$, $D^0$, $D_s$ production.

I will also rely heavily on a review article compiled for charm hadroproduction [hep-ph/0609101 - 11 Sep 2006, to appear in Physics Reports]. This article is used for total cross sections only.

I estimate the total relative systematic error, assuming the errors can be added in quadrature.

Total Cross Sections

There is little data on total cross sections for $D^+$, $D^0$, $D_s$ production and I could find nothing significant for $\Lambda_c$ total cross section. The data is spread over a range of beam energies, but there is a “cluster” of 4 experiments at or near 800 GeV. The evolution of the cross section with $\sqrt{s}$ is reasonably understood and is due to kinematic effects of the bare charm mass (~1.5 GeV) giving a dependence very similar to the tau threshold curve. I believe the uncertainty of fitting this curve (with Pythia, etc.) to data at all energies is definitely smaller than the error in any single experiment and comparable to the error for the four high-energy data sets combined in quadrature. All data reported in the table of cross sections is valid for all $x_F$ (not just $x_F>0$).

$D^0$, $D^\pm$

The production of charged and neutral D mesons (D-bar is not distinguished here) is summarized in Table 4 of Ref. [1]. In the following table, I include the Hera-B point by evolving it down to 800 GeV using the Pythia function shown in Fig. 12 Ref. [1] to get a factor of 0.89.

<table>
<thead>
<tr>
<th>Exp</th>
<th>$\sigma(D^0)$</th>
<th>$\sigma(D^\pm)$</th>
<th>$\sigma_{cc}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hera B</td>
<td>40.3±5.3</td>
<td>19.4±3.3</td>
<td>39.5±4.2</td>
</tr>
<tr>
<td>E653</td>
<td>43±14</td>
<td>37±15</td>
<td>48±13</td>
</tr>
<tr>
<td>E743</td>
<td>22±11</td>
<td>26±8</td>
<td>29±8</td>
</tr>
<tr>
<td>E789</td>
<td>17±3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wtd. Avg</td>
<td>37.5±4.5*</td>
<td>21.0±3.0</td>
<td>38.0±3.6</td>
</tr>
</tbody>
</table>

The weighted average of the $D^0$ cross section excludes E789. The Pythia fit gives a central value of 39(±2)µb for $D^0$ and a central value of 20(±1)µb.

We conclude that despite the meager number of experiments, the data (exclude E789) is consistent and compatible with lower energy data as seen in the Pythia fit. This is in large part due to the recent Hera-B results.
I would propose using the Pythia estimate of 39μb for D⁰ and 21μb for D⁺, or equivalently, a D⁺/D⁰ ratio of 0.51, which is the same as the ratio from the data, 0.51±0.06, seen below. Note that for πN this ratio is 0.37±0.03. For the Hera-B measurements alone, this ratio is 0.40±0.07, compatible with πN.

Figure . Average D⁺/D⁰ ratio for pN data.

Figure . D⁰ production cross sections assuming normal errors. Very poor agreement between the expected (~40μb) and E789. The Pythia fit gives between 37 - 41μb, with E789 not used.

Ds

The total production cross section for Ds is not as well measured as non-strange charm, there are no reliable cross sections from pN data that I could find. We must rely on the Ds/D⁰ ratio as observed in πN data. This ratio is taken from πN data only.
and is found to be $D_s/D^0 = 0.203\pm0.031$. This gives $7.9\pm1.2\mu b$. The $\pi N$ data is not optimal for estimating $p N$ interactions as there may be leading quark effects in reactions (see Table below).

$\Lambda_c$
There are no reliable measurements for the $\Lambda_c$ cross section that I could find. I think the best estimator of $\sigma(\Lambda_c)$ is to assume that it accounts for all of the remaining inclusive cross section of the total, given as $\sigma(cc)$. The sum of the inclusive $D^0$, $D^\pm$ and $D_s$ cross sections is $39+21+8 = 68\mu b$ (with error $\sim5\mu b$). The average value of $\sigma(cc)$ is $38\mu b$, or equivalently, $76\mu b$ of inclusive charm. Therefore, I take the $\Lambda_c$ cross section as $8\pm5\mu b$.

**Differential Cross Sections**

$xf$ Dependence
We use the “standard” parameterization in $xf$ (i.e. $(1-|x|)^n$) for the following compilation of data from SELEX. “Leading particle” effects are important in charm production, i.e. one or more quarks in common with the content of the incident beam. So for the proton beam, $uud$, $n(\Lambda_c)$ is very different for the particle and antiparticle for both $p N$ and $\Sigma N$ production.

<table>
<thead>
<tr>
<th>Beam</th>
<th>Particle</th>
<th>$n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p$</td>
<td>$D^0$</td>
<td>$5.9\pm0.5$</td>
</tr>
<tr>
<td></td>
<td>$D^0\text{bar}$</td>
<td>$7.3\pm0.3$</td>
</tr>
<tr>
<td></td>
<td>$D^+$</td>
<td>$4.4\pm0.4$</td>
</tr>
<tr>
<td></td>
<td>$D^-$</td>
<td>$4.7\pm0.4$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Beam</th>
<th>Particle</th>
<th>$n$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\Lambda_c^+$</td>
<td>$2.2\pm0.3$</td>
</tr>
<tr>
<td></td>
<td>$\Lambda_c^-$</td>
<td>$9\pm7$</td>
</tr>
<tr>
<td></td>
<td>$\pi$</td>
<td>$3.7\pm0.4$</td>
</tr>
<tr>
<td></td>
<td>$D^0$</td>
<td>$5.0\pm0.4$</td>
</tr>
<tr>
<td></td>
<td>$D^0\text{bar}$</td>
<td>$2.5\pm0.3$</td>
</tr>
<tr>
<td></td>
<td>$D^+$</td>
<td>$3.6\pm0.3$</td>
</tr>
<tr>
<td></td>
<td>$\Lambda_c^+$</td>
<td>$2.7\pm0.4$</td>
</tr>
<tr>
<td></td>
<td>$\Lambda_c^-$</td>
<td>$2.2\pm0.8$</td>
</tr>
<tr>
<td></td>
<td>$\Sigma^-$</td>
<td>$6.2\pm0.3$</td>
</tr>
<tr>
<td></td>
<td>$D^0$</td>
<td>$7.3\pm0.3$</td>
</tr>
<tr>
<td></td>
<td>$D^0\text{bar}$</td>
<td>$5.0\pm0.2$</td>
</tr>
<tr>
<td></td>
<td>$D^+$</td>
<td>$4.7\pm0.2$</td>
</tr>
<tr>
<td></td>
<td>$D^-$</td>
<td>$3.9\pm0.4$</td>
</tr>
<tr>
<td></td>
<td>$D_s^+$</td>
<td>$8.0\pm0.8$</td>
</tr>
<tr>
<td></td>
<td>$\Lambda_c^+$</td>
<td>$2.5\pm0.2$</td>
</tr>
<tr>
<td></td>
<td>$\Lambda_c^-$</td>
<td>$6.8\pm1.1$</td>
</tr>
</tbody>
</table>
Some general observations:

1. when leading effects are not expected in $p$ or $\Sigma$ beams, the $n$ values are within error, the same

2. $D_+\bar{c}$ is favored by $\Sigma^-$ beam but $D_+\bar{c}$ is not

3. $\Lambda_c$ production is very asymmetric with $p$ or $\Sigma$ beams

A conservative assumption is that we use the $pN$ values from the table, but in addition:

- use $n(D_+\bar{c}) = 8.0\pm0.8$ from $\Sigma$ production
- use $n(\Lambda_c) = 6.8\pm1.1$ from $\Sigma$ production

$p_T$ Dependence

The SELEX data do not favor a simple exponential in $p_T^2$. There is a break around 2 (GeV/$c$)$^2$. A simple exponential fit over the $p_T$ range 0 to 2.5 GeV/$c$ would give a lower value for $b$, around 0.9. Pion data favor a simple exponential fit.

$A$ Dependence

Open charm and $J/\psi$ production favor a value of $\alpha$ very near 1.0. In Hera-B, $J/\psi$ has $\alpha = 0.969\pm0.003$, consistent with values we have used [0.987±0.026 (Emily and Reinhard)]

Secondary Charm Production

The data above and models used in the MC are only for direct charm production. The contribution of secondary production is taken to be $0.075\pm0.033$ (Patrick) for the “expected fraction of identified interactions”. I assume this applies to $\nu_{\bar{c},\mu}$. For $\nu_{\tau}$, I correct this fraction by the ratio of the kinematic factor for the average neutrino energy of 56 and 33 GeV, primary and secondary, respectively. This gives $K(33)/K(56) = 0.74$, so the fraction of secondary $\nu_{\tau}$ is estimated to be $0.056\pm0.024$. 

![Figure](image.png)
Systematics in Number of Neutrino Interactions

There are many parameters that affect the number of neutrino interactions in DONuT, giving rise to systematic uncertainties. I will classify them into two groups: (1) parameter variations giving a linear (or nearly linear) change to $N_{\text{int}}$ and (2) parameters that kinematics of charm and must be simulated with the Monte Carlo.

Type 1 Linear.

Varying the total cross sections for charm is obviously a linear effect. The $A$ dependence over the range $\alpha = 0.95$ to 1.03 is also very close to linear, and is just a simple multiplier, like total cross section.

The total uncertainty for charm production is found by adding the errors for each process:

$$\delta[\sigma(\text{cc})] = 4.5 \oplus 3.6 \oplus 1.2 \oplus 5 = 7.7$$

so the total relative error is

$$7.7 / (37.5+21.+7.9+8.) = 0.10$$

The relative error for the $A$ dependence is 0.026.

We assume also that branching fractions give a linear effect, although this is not exactly true. For $\nu_\tau$ production the relative error is $0.015/0.064 = 0.23$ (the error in the $\tau \rightarrow e / \mu$ branching fraction can be ignored). For $\nu_{e,\mu}$ production I get the relative error in branching fraction to be 0.16 (I assume equal for $e$ and $\mu$.)

The number of neutrino interactions from secondary interactions has large uncertainty, but the fraction of events is relatively small. We take the uncertainty in the number of $\nu_{e,\mu}$ interactions to be the error 0.033, and the uncertainty in the number of $\nu_\tau$ interactions to be 0.024.

This gives a total relative uncertainty in the number of $\nu_{e,\mu}$ interactions to be $0.10 \oplus 0.026 \oplus 0.033 \oplus 0.16 = 0.19$ for “linear” types of errors. For $\nu_\tau$ this is $0.10 \oplus 0.026 \oplus 0.024 \oplus 0.23 = 0.25$.

Note that for the relative cross section results the total relative error we can neglect the $A$ dependence error and the total cross section contribution will be dominated by error in $D_s$ production. For the relative cross section error:

$$[\delta \sigma(D_s)] \oplus [\delta \text{BR}(D_s)] \oplus [\delta \text{BR}(D^{0\pm})]$$

$$= 0.15 \oplus 0.23 \oplus 0.16 = 0.32$$

This is larger than $0.19 \oplus 0.25 = 0.31$ since the relative error of $D_s$ alone is larger compared with the error in $\sigma(\text{cc})$.

Type 2 Kinematics

The $x_F$ dependence is a large kinematic effect on the neutrino spectrum and is simulated in the Monte Carlo to calculate the variation in interaction yield with $n$.

In addition, we need to roughly know how a particle / anti-particle asymmetry uncertainty affects $N_{\text{int}}$. Although the ratio of $\mu^+$ to $\mu^-$ events in CC$\mu$ interactions limits the likely variation.
For \( \nu_\tau \) production we assume the only source is \( D_s \) decays. Varying the parameter \( n \) changes the yield in a roughly linear way, as shown in the accompanying figure. Using \( n = 8.0 \pm 0.8 \) gives the product of \( f^{<\Sigma EKT>} \) = 2.23\( \pm 0.40 \). So the relative error in differential cross section is 0.18. But we must consider a "\( \Delta n \)" error (difference in \( n \) between charm particle and its anti-particle) where possible leading particle effects are taken into account. Except for \( \Lambda_c \), the SELEX production data favors \( \Delta n < 2 \). Our data also tend to favor small \( \Delta n \) (see figure). If we restrict the allowable range of \( \Delta n < 2 \) then the relative change in \( f^{<KE>} \) is about 0.26 almost independent of \( n \). If we use \( \Delta n < 1 \) then the ratio is 0.13.

Then, in summary, the systematic uncertainties in the differential charm cross sections give a total relative error (using \( \Delta n = 2 \)):

\[
0.18 \oplus 0.26 = 0.32
\]

It does not seem reasonable that these cancel with the relative cross section measurement, since the \( D_s \) differential behavior is not dependent on the \( D^0 \) or \( D^\pm \) cross section.

If I put this all together, in quadrature, for the relative cross sections (geometric addition is distributive)

\[
0.32 \oplus 0.32 = 0.45
\]

Figure. The red diamonds are MC estimates for the number of \( \nu_\tau \) interactions as a function of \( n \). Also shown is the effect of fixing \( D_s^- \) (at \( n = 6 \)) and changing \( D_s^+ \) production (\( n = 7, 8 \)).
Figure 1. Variation of interactions as a function of $b$. The effect is small, about 5% over the range of possible values.

Figure 2. MC results for the ratio of the number of $\mu^+$ to $\mu^-$ events for various values of $n$. For each data point color, the value of the anti-particle (D0-bar, D-, Ds-) is fixed. Notice that the higher points are those for $\Delta n=0$ and is favored by the experiment. The blue shaded region is one standard deviation.
References:


