Progress report on ν_τ CC - NC scattering event classification (very preliminary) & Event Location status

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Outline

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ANN probability

2 Theorems related with MLP (Multi Layer Perceptron)

- Any continuous function can be approximated to any accuracy by a linear combination of sigmoids

K.Hornik et. al Neural Networks, Vol.2, pp 359-366 (1989)

- A MLP trained with f(x)=1 for Signal and f(x)=0 for Background events, approximates the a posteriori class probabilities P(S/x) (x =feature vector) of the Signal, if one knows x.

(D.W.Ruck et al. 'The MLP as an Approximation to a Bayes Optimal Discriminant Function, IEEE Trans. On Neural Networks, Vol. 1, pp 296-298 (1990)
M.D.Richard and R.P.Lippman 'Neural Network Classifiers estimate the Bayesian a posteriori probabilities', Neural Computation, Vol.3 pp 461-483 (1991))

ANN probability (cont.)

ANN analysis : Minimization of an Error (Cost) Function

$$\mathbf{E_N} = \frac{1}{\mathbf{N}}\sum_{\mathbf{N}}(\mathbf{f}(\mathbf{x_i},\mathbf{w}) - \mathbf{t_i})^2, \text{ w = weights, f(x, w) = ANN output, x = feature vector}$$

$$\text{t = desired ANN output (1 Signal \& 0 background)}$$

$$E_N = \frac{\mathbf{N_S}}{\mathbf{N}}\frac{1}{\mathbf{N_S}}\sum_{\mathbf{S}}(\mathbf{f}-1)^2 + \frac{\mathbf{N_B}}{\mathbf{N}}\frac{1}{\mathbf{N_B}}\sum_{\mathbf{B}}(\mathbf{f}-0)^2$$

$$\lim_{\mathbf{N},\mathbf{N_S},\mathbf{N_B}\to\infty}\mathbf{E_N} = \lim_{\mathbf{N},\mathbf{N_S},\mathbf{N_B}\to\infty}\left(\frac{\mathbf{N_S}}{\mathbf{N}}\frac{1}{\mathbf{N_S}}\sum_{\mathbf{S}}(\mathbf{f}-1)^2 + \frac{\mathbf{N_B}}{\mathbf{N}}\frac{1}{\mathbf{N_B}}\sum_{\mathbf{B}}(\mathbf{f}-0)^2\right)$$

$$\text{but} \lim_{\mathbf{N},\mathbf{N_S}\to\infty}\frac{\mathbf{N_S}}{\mathbf{N}} = \mathbf{P(S)} \;\&\; \lim_{\mathbf{N},\mathbf{N_B}\to\infty}\frac{\mathbf{N_B}}{\mathbf{N}} = \mathbf{P(B)}$$

$$\text{and} \lim_{\mathbf{N_S}\to\infty}\frac{1}{\mathbf{N_S}}(\mathbf{f}-\mathbf{s})^2 = \int(\mathbf{f}-\mathbf{s})^2\mathbf{P}(\mathbf{x}/\mathbf{S})\mathbf{dx} \;\ldots..$$

$$\ldots\ldots f = P(S/x)$$

The ANN output is the Bayes a posteriori probability & in the proof no special assumption has been made on the a priori P(S) and P(B) probabilities (absolute normalization) which are usually selected equal (number of Signal = Number of Background events)

ANN probability (cont.)

- Bayesian a posteriori probability :

$$P(S/x) = \frac{P(x/S) * P(S)}{(P(S) * P(x/S) + P(B) * P(x/B)}$$

$P(S)$ = a priori signal probabilty — $P(x/S)$ = Signal probabilty density function

$P(B)$ = a priori background probabilty — $P(x/B)$ = Background probabilty density function

$\Rightarrow$ ANN output : P(S/x)

$\Rightarrow$ ANN training examples : P(x/S) & P(x/B)

$\Rightarrow$ ANN number of Signal Training Examples P(S)

$\Rightarrow$ ANN number of Background Training Examples P(B)

- The MLP (ann) analysis and the Maximum Likelihood Method (Bayes Classifier) are equivalent.

(c_{11} c_{22} = cost for making the correct decision & c_{12} c_{21} = cost for making the wrong decision)

$$\Lambda(x) = \frac{P(x/S)}{P(x/B)} \;\&\; \xi = \frac{P(S)(c_{12} - c_{11})}{P(B)(c_{21} - c_{22})}$$

$$\text{if } c_{11} = c_{22} = 0 \;\&\; c_{12} = c_{21} \Rightarrow$$

$$\Lambda(x) > \xi \Leftrightarrow \frac{P(x/S)}{P(x/B)} > \frac{P(B)}{P(S)} \Leftrightarrow P(x/S) * P(S) > P(x/B) * P(B) \Leftrightarrow$$

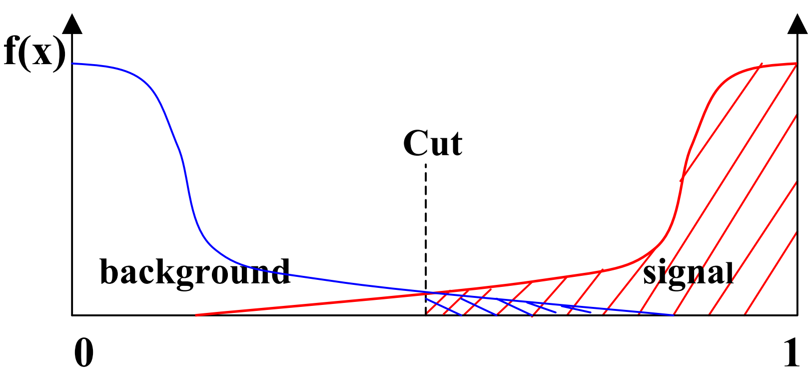
$$\Leftrightarrow \frac{P(x/S) * P(S)}{P(x)} > \frac{P(x/B) * P(B)}{P(x)} \Leftrightarrow P(S/x) > P(B/x) \Leftrightarrow$$

$$\Leftrightarrow P(S/x) > (1 - P(S/x)) \Leftrightarrow P(S/x) > 0.5$$

ANN & Bayes probability of Error

- The ANN and Maximum Likelihood Method (Bayes Classifier) use a threshold above which consider the events as signals or backgrounds (usually 0.5 for the ANN analysis).

- Apart for the event probability (ANN output) there is another important probability : The ***Misclassification Probability P_e***



$$\mathbf{P_e = P(S) * P(e/S) + P(B) * P(e/B)}$$

$\mathbf{P(S)}$ & $\mathbf{P(B)}$: a priori probabilities of S and B

e : Set of erroneous classifications

P(S) = % of Signal Events P(B) = % of Signal Events

$P(e/S) = N_{back,> cut} / N_{all > cut}$ $P(e/B) = N_{sig.,< cut} / N_{all < cut}$

ANN for tau CC - NC scattering

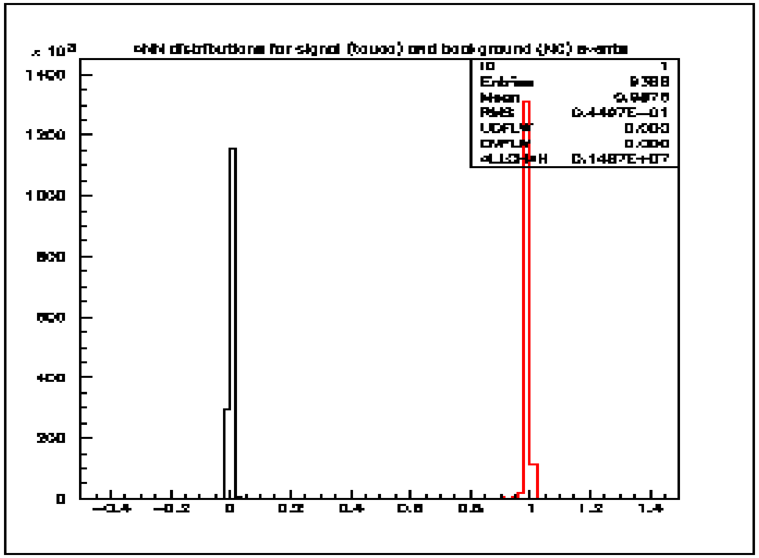
- **Goal :** To separate tau CC interactions from NC scattering interactions with the use of ANNs

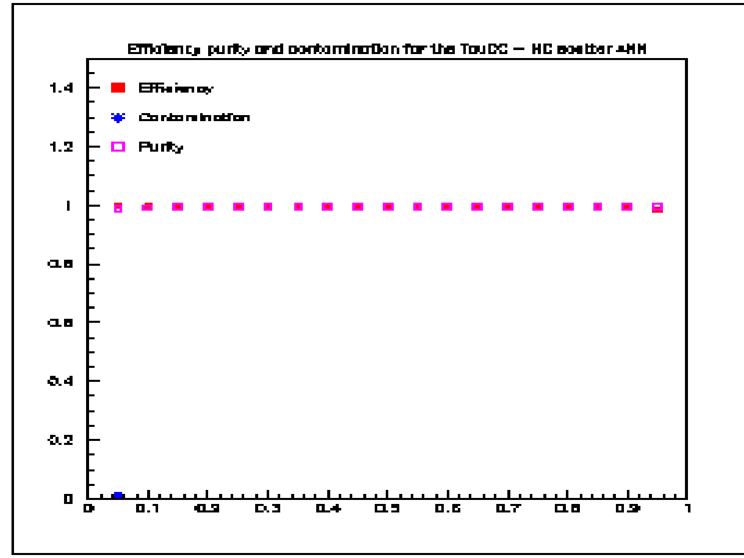
- **Input Variables :**
  - **Daughter Momentum**
  - **Parent flight**
  - **Parent angle**
  - **Daughter angle**
  - **(Φ angle has not been used. How is it estimated??)**

- **Training Set :**
  - 20000 Tau CC interactions
  - 20000 NC scattering interactions

- **No cuts have been applied to any of these parameters. Therefore all events have been considered (below and above P_T=250 MeV/c)**

ANN Performance





- ANN structure 4 - 5 - 1 (Other architectures with 2 Hidden Layers i.e 4-5-2-1 give very similar results)
- **Cut@0.5 :**
  - **Efficiency = 99.8 %**
  - **Purity = 99.6 %**
  - **Contamination = 0.4 %**
  - **P(error) = 0.4 %**

Relative ANN Weights

Momentum = 42.6

Daughter Angle = 24.4

Parent Flight = 16.8

Lepton Angle = 16.2

- ANN Performance is good & All variables seem to be important (all the weights are relatively high)

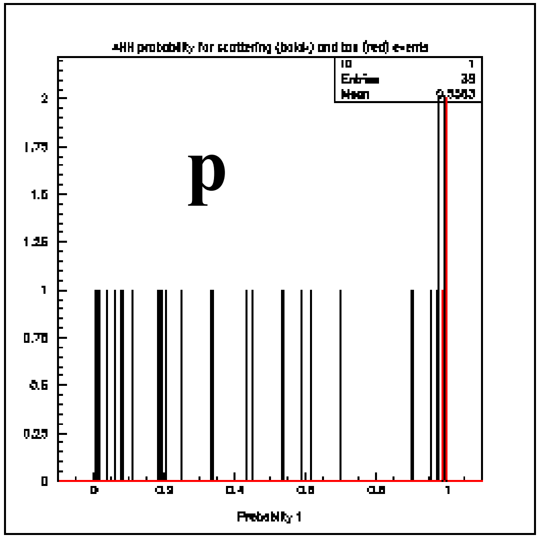
ANN Results on PhaseI&II kinks & tau CC events

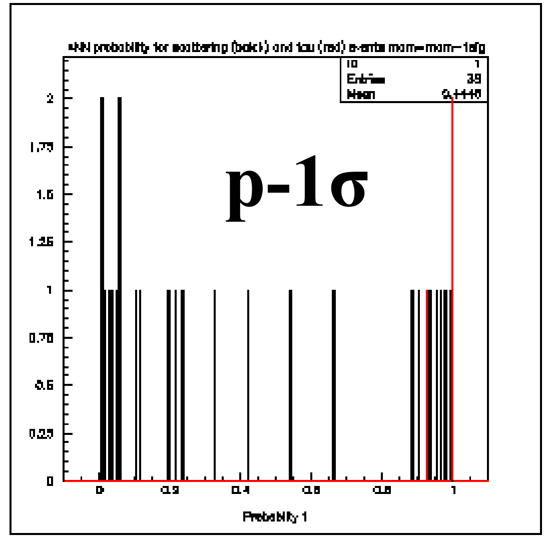
- We computed the ANN output on 38 kinks (supplied to us by Nonaka) and on the 4 tau CC events. 9/38 are Phase I kinks and the remaining Phase II.
- We repeated this procedure for the central value of the momentum and for 1 , 2, 2.5 and 3 sigma below.

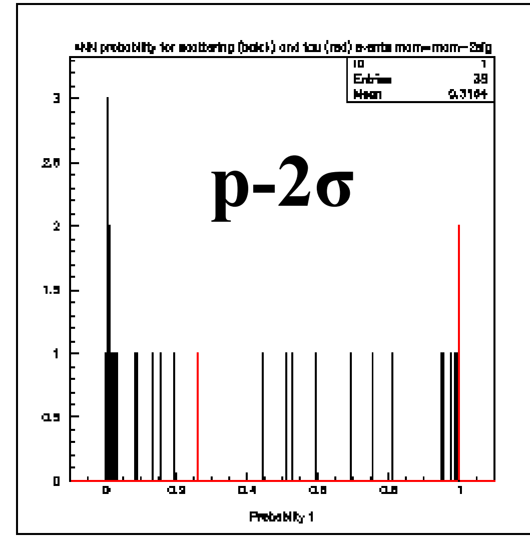
Kinks with ANN probability > 0.5	Momentum
21	$p_{meas.}$
15	p_{meas} -1σ
11	p_{meas} -2σ
6	p_{meas} -2.5σ
2	p_{meas} -3σ

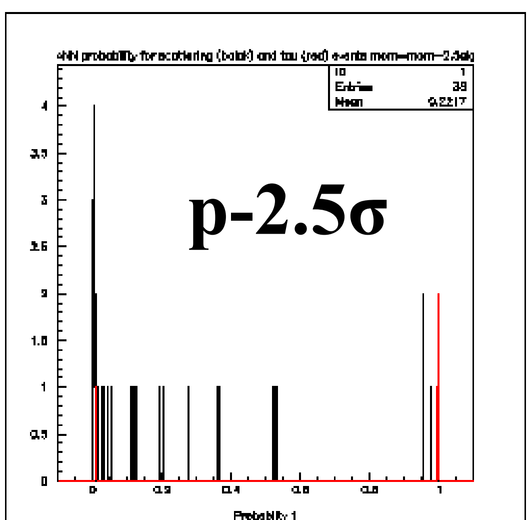
Taus with ANN probability > 0.5	Momentum
4 (0.987-0.998-0.998-0.999)	$p_{meas.}$
4 (0.927-0.998-0.998-0.998)	p_{meas} -1σ
3 (0.261-0.997-0.997-0.998)	p_{meas} -2σ
3 (0.010-0.994-0.997-0.997)	p_{meas} -2.5σ
3 (0.000-0.951-0.997-0.996)	p_{meas} -3σ

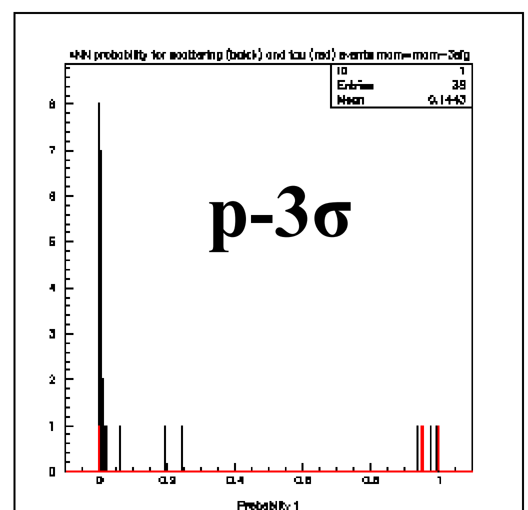
ANN Results on PhaseI&II kinks & tau CC events (cont.)











ANN Results on PhaseI&II kinks & tau CC events (cont.)

- Three of the 4 tau events have probability > 0.98 for momentum p, p-1σ, p-2σ and p-2.5σ and > 0.95 for p-3σ.
- The number of Phase I kinks with this characteristic are zero and the number of Phase II kinks are :
  - 9 (p) 4 (p-1σ) 3(p-2σ) 1(p-2.5σ) and 1(p-3σ)

ANN Results on PhaseI&II kinks & tau CC events (cont.)

2 Kinks that their probability is still > 0.5 for momentum p-3σ

event	RUN	MOMD	err+	err-	LEPANG	THETAD	FLIGHT	prob.
3111	06248	2.90	0.90	0.60	0.0554	0.0719	2780.3	0.937
3182	10057	3.70	0.00	0.60*	0.1487	0.1280	0025.1	0.997

* It had 0 error so I assigned a logical value.

4 Tau events

event	RUN	MOMD	err+	err-	LEPANG	THETAD	FLIGHT	prob.
3263	25102	2.00	2.20	0.70	0.19036	0.1534	1815.1	0.987
3024	30175	2.90	1.50	0.80	0.02786	0.1206	4504.8	0.997
3039	01910	4.60	1.60	0.90	0.06530	0.0900	276.5	0.997
3333	17665	21.40	14.40	6.00	0.01538	0.0181	564.6	0.999

ANN Summary

- The ANN output can be interpreted as the Bayesian a posteriori probability of Signal knowing x : P(S/x)

- The ANN tau CC - NC scattering performance is quite satisfactory & shows that all four ANN parameters are important.

- The implementation of the ANN on the 38 kinks yields results that we need to understand.
  - a) How events with high Pt and the other three parameters close to the ones expected for tau's are categorized as NC scattering.

event	RUN	MOMD	err+	err-	LEPANG	THETAD	FLIGHT
3020	**24151**	**8.80**	**7.10**	**2.90**	**0.0736**	**0.0686**	**2675.9**

  - b) What is the criterion apart from Pt with which NC scattering events are rejected.

- The implementation of the ANN on the four tau events shows that initially all the events have high probabilities of being taus. When the momentum is lowered by more that 1 σ one of the events is assigned a very low probability.

Event location status & Decay Search

- By remaking the Phase II list we found that out of 227 events 50 are not located by Bruce or us. (Byron has the list we prepared so we can cross-check and end up with the final event location status)

- We are attempting event location of new events.

- We are proccesing all located events of Phase II with the decay search code in order to make sure that we get the same results with Nonaka.

On going work

- We are continuing to study the tau CC - NC scattering classification with the ANN approach.

- We are attempting Event location and Decay Search